

# Kaluza-Klein gluon and $b$ -jet forward-backward asymmetry

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The forward-backward asymmetry of  $b$ -quark jets on the  $Z$ -pole measured at LEP/SLD experiments shows us  $-2.8\sigma$  deviation from the Standard Model (SM) prediction. We examine a possibility of Kaluza-Klein (KK) gluon to explain the  $A_{FB}^b$  data in a scenario based on the warped extra dimension model by Randall and Sundrum. In this scenario, the KK gluon strongly couples to  $b$ -quark by an appropriate choice of the bulk quark mass terms. We find that the  $A_{FB}^b$  data could be explained if the KK gluon mass is few hundred GeV. Constraints on our scenario from the hadron collider experiments are discussed.

Standard Model (SM) of particle physics has shown a good agreement with the results of electroweak experiments performed on the  $Z$ -pole [1], except for the forward-backward (FB) asymmetry of  $b$ -quark jets ( $A_{FB}^b$ ). The experimental data of  $A_{FB}^b$  is [1]

$$A_{FB}^b(\text{exp.}) = 0.0992 \pm 0.0016, \quad (1)$$

while the SM prediction is [1]

$$A_{FB}^b(\text{SM}) = 0.1037, \quad (2)$$

for the best fit of the SM. From (1) and (2) we find about  $-2.8\sigma$  deviation. Although it might be caused due to a lack of our understanding of the  $b$ -jet data as discussed in [2], in this article we would like to examine a possibility of the deviation as an implication of new physics beyond the SM. The electroweak observables at the  $Z$ -pole experiments can be expressed in terms of the effective coupling  $g_\alpha^f$  which denotes the interaction between  $Z$  and  $f_\alpha$ , where  $f$  denotes fermion species and  $\alpha (= L, R)$  is their chirality. The radiative corrections to  $g_\alpha^f$  consist of the gauge boson propagator corrections (so called the oblique corrections) which are often parametrized by  $S$  and  $T$  [3], and the  $Zff$  vertex correction  $\Delta g_\alpha^f$ . When the oblique correction is dominated by SM, the new physics contribution to the FB asymmetry,  $A_{FB}^b(\text{NP})$ , is given as follows [4]:

$$A_{FB}^b(\text{NP}) = A_{FB}^b(\text{SM}) - 0.0326\Delta g_L^b - 0.1789\Delta g_R^b. \quad (3)$$

It is convenient to define the additional new physics contribution to  $A_{FB}^b$  in the unit of  $10^{-4}$

$$\delta A_{FB}^b \equiv (A_{FB}^b(\text{NP}) - A_{FB}^b(\text{SM})) \times 10^{+4}. \quad (4)$$

The present experimental data (1) constrains the new physics contribution (4) as

$$\delta A_{FB}^b = -45 \pm 16, \quad (5)$$

at the  $1\sigma$  level.

Several attempts have been done to explain (5) based on various new physics models – *e.g.*, supersymmetry [5], extended gauge symmetry [6], extra vector-like quarks [7], etc. Kaluza-Klein (KK) particles of the SM fields in a variant of warped extra dimension model by Randall and Sundrum (RS) [8] is also one of the possibilities. In this model, the KK modes of gauge bosons and fermions contribute to both the oblique and  $Zbb$  vertex corrections. It has been shown that the KK modes of the electroweak gauge bosons give significantly large contribution to the oblique parameters since there is no custodial symmetry in the bulk. As a result, the scale of KK mode  $\Lambda_{\text{KK}}$  is strongly constrained from the electroweak data, say  $\Lambda_{\text{KK}} > O(10^{2-3}\text{TeV})$ , which leads to unwanted hierarchy between the electroweak scale  $\Lambda_{\text{EW}} \sim O(m_W)$  and  $\Lambda_{\text{KK}}$  [9, 10]. Such a constraint could be somewhat lowered to  $O(\text{TeV})$  by introducing the custodial symmetry in the bulk, or additional contribution from the bulk SM fermions [11, 12]. Taking account of these constraints, the  $A_{FB}^b$  puzzle has been studied in a variant of RS model, *e.g.*, in refs. [13, 14], where the deviation of  $A_{FB}^b$  is explained by the mixing of the  $Z$  boson and its KK states. On the other hand, since the gluon does not couple to the electroweak gauge bosons directly, the KK modes of the gluon are not constrained from the oblique parameters. This motivates us to examine a possibility to explain  $\delta A_{FB}^b$  (5) by KK gluons without conflicting the other electroweak precision data.

In this article, we would like to study the KK gluon contribution to  $A_{FB}^b$  in the warped extra dimension model. It is known that, in warped extra dimension model, the 4D effective coupling of KK gluon and fermions is determined by the overlap of their wavefunctions in the fifth dimension. With an appropriate choice of the bulk quark mass terms, the coupling of the KK gluon to the  $b$ -quark could sizably enhanced while the others are suppressed. Then, the 1-loop KK gluon exchange could shift the  $Zbb$  vertex correction  $\Delta g_\alpha^b$  without any shift to  $\Delta g_\alpha^f (f \neq b)$ . We find that, in this scenario, the puzzle of  $A_{FB}^b$  could be resolved when the

1st KK gluon mass is few hundred GeV. As mentioned above, the KK scale is constrained to be  $O(\text{TeV})$  taking account of the contributions of KK  $W, Z$  bosons to oblique parameters. In this case the KK gluon mass also must be  $O(\text{TeV})$  which cannot give sizable correction to  $Zbb$  vertex. Therefore our scenario of relatively light KK gluon faces difficulty in models which has been known so far. However it is worth studying the QCD corrections to the  $Zbb$  vertex independently from the structure of electroweak sector in warped extra dimension model.

Phenomenology of the KK gluon has been studied in, e.g., ref. [15], focusing on the production and decay at LHC. The KK gluon in [15] is, however, assumed to couple strongly to the  $t_R$  quark and contribution to  $\delta A_{FB}^b$  is not considered.

Let us briefly review the interactions of the KK gauge boson to fermions in the warped extra dimension model. The model consists of a non-factorizable geometry on  $AdS_5$  with metric

$$ds^2 = e^{-2k|y|} \eta_{\mu\nu} dx^\mu dx^\nu - dy^2, \quad (6)$$

where  $y$  is the coordinate of the fifth dimension and  $k$  denotes the  $AdS_5$  curvature. Two 3-branes – “Planck” and “TeV” branes – locate at fixed points of  $S^1/Z_2$  orbifold,  $y = 0$  and  $y = \pi r_c$ , respectively. The hierarchy between the Planck and Electroweak scales can be explained reasonably when  $kr_c \approx 11$ . In general, if a SM fermion  $\Psi$  can propagate into the bulk, there is a 5D mass term  $m_\Psi \bar{\Psi} \Psi$  in the 5D action without breaking the SM gauge symmetry. As shown in [16], the 5D fermion mass  $m_\Psi$  can be expressed as  $m_\Psi = \nu_\Psi k \epsilon(y)$ , where  $\epsilon(y)$  is  $+1$  for  $y > 0$  while  $-1$  for  $y < 0$  to make the mass term to be  $Z_2$ -even. The wavefunction of the zero mode fermion, then, has the peak toward the Planck brane for  $\nu_\Psi < -1/2$  and toward the TeV brane for  $\nu_\Psi > -1/2$ . The effective 4D interaction of a fermion  $f^{(n)}$  and a gauge boson  $A_\mu^{(m)}$  can be obtained by integrating the 5D action over  $y$ , where  $f^{(n)}$  and  $A_\mu^{(m)}$  are the 4D KK modes of the 5D fermion  $\Psi$  and gauge boson  $A_M$ , respectively, and  $n, m$  are positive integer. Then, the effective coupling of the zero mode fermion  $f (= f^{(0)})$  and KK gauge boson  $A^{(n)}$  is given as a function of the parameter  $\nu_\Psi$ . The generic formula of  $g^{ffA^{(n)}}$  can be found, e.g., in ref. [10]. For  $n = 1$ , the coupling  $g^{ffA^{(1)}}$  can be expanded in terms of  $\nu_\Psi$  as follows:

$$g^{ffA^{(1)}} \approx g_{\text{SM}} \times \begin{cases} -0.2 & (\nu_\Psi < -0.5) \\ 4.0 + 5.2\nu_\Psi - 4.6\nu_\Psi^2 + 2.1\nu_\Psi^3 & (\nu_\Psi > -0.5), \end{cases} \quad (7)$$

where  $g_{\text{SM}}$  denotes the SM gauge coupling in 4D. In Fig. 1 we depict a ratio  $g^{ffA^{(1)}}/g_{\text{SM}}$  as a function of  $\nu_\Psi$ . We find that the coupling  $g^{ffA^{(1)}}$  is enhanced significantly for  $\nu_\Psi \gtrsim -0.4$  as compared to the SM gauge coupling  $g_{\text{SM}}$ . On the other hand, the coupling  $g^{ffA^{(n)}}$  is highly suppressed for  $\nu_\Psi \lesssim -0.5$ . The couplings of the higher

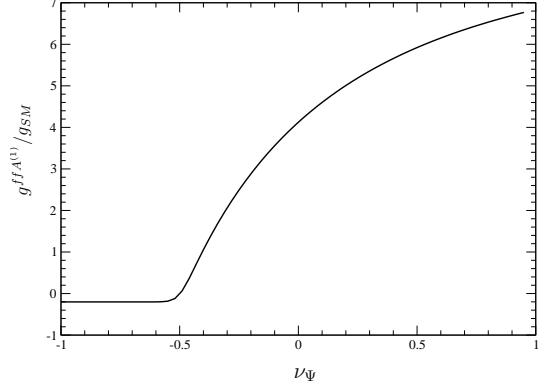


FIG. 1: The ratio of 4D effective coupling of the 1st KK mode of the gauge boson to the fermion,  $g^{ffA^{(1)}}$ , and the SM gauge coupling  $g_{\text{SM}}$  as a function of the parameter  $\nu_\Psi$ .

KK mode of gauge boson with fermions are also highly suppressed when  $\nu_\Psi \lesssim -0.5$ . In the literature, the parameter  $\nu_\Psi$  is considered as an origin of the hierarchy of 4D Yukawa couplings. The values of  $\nu_\Psi$  for each flavor are constrained to reproduce the hierarchy of 4D Yukawa couplings[12]. In our study, however, we take  $\nu_\Psi$  as a model parameters to explain the  $A_{FB}^b$  data.

Next, we examine the QCD correction to the  $Zbb$  vertex due to the exchange of the KK gluon  $g^{(n)}$  and  $b^{(m)}$ -quarks. In our study, we consider possibilities that the  $b (= b^{(0)})$ -quarks strongly couple to  $g^{(n)}$ , which corresponds to cases  $\nu_{Q_{3L}} \text{ or } \nu_{b_R} \gtrsim -0.5$ , where  $Q_{3L} = (t_L, b_L)$ . We set  $\nu_{\text{others}} \lesssim -0.5$  for the other light quarks so that those couplings to  $g^{(n)}$  are neglected. We do not consider the  $t$ -quark in the following because it does not contribute to  $Zff$  vertex through the QCD correction. Then, the contributions of KK gluon to  $Zbb$  vertex are determined by the  $\nu$ -parameters for  $b_L, b_R$  and the KK gluon mass,  $m_{g^{(1)}}$ . From phenomenological point of view, it is useful to introduce a new parameter  $\xi_\alpha \equiv g^{b_\alpha b_\alpha g^{(1)}}/g_s$ , instead of the  $\nu$ -parameters.

The Feynman diagrams of  $Zbb$  vertex via the KK gluon exchange are shown in Fig. 2. The vertex correction

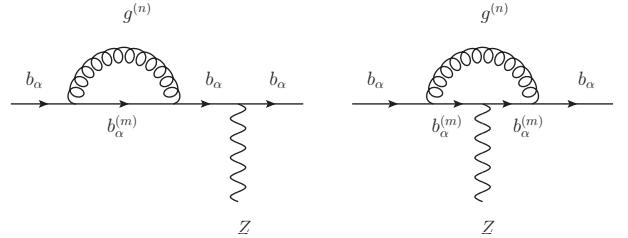


FIG. 2: The Feynman diagrams of 1-loop  $Zbb$  vertex.

$\Delta g_\alpha^b$  ( $\alpha = L, R$ ) is given as follows:

$$\Delta g_\alpha^b = \frac{1}{\sqrt{4\sqrt{2}G_F m_Z^2}} (g_\alpha^{bbZ} \Sigma'(0) - \Gamma_{b_\alpha}(m_Z^2)), \quad (8)$$

where  $\Sigma'(0)$  is the derivative of the self energy function of the external  $b$ -quark, whose mass is neglected. The scalar function  $\Gamma_{b_\alpha}(m_Z^2)$  is the three point function of the  $Zb_\alpha b_\alpha$  vertex at the momentum transfer  $q^2 = m_Z^2$ . The coupling of the  $Z$ -boson to  $b_\alpha$  quarks is denoted by  $g_\alpha^{bbZ}$ . We note that the ultra violet divergences are cancelled between the self energy and vertex diagrams from each KK state. However, the 1-loop corrections become infinite when one takes the sum of the finite contributions from whole KK towers. We, therefore, need to introduce a cut-off scale  $\Lambda$  to restrict the number of KK modes.

The Naïve Dimensional Analysis (NDA)[17, 18] has been adopted to determine the cut-off scale  $\Lambda$ . In NDA, the cut-off scale  $\Lambda$  is interpreted as an upper limit of energy scale in which a theory is perturbative. However, NDA tells us that the cut-off scale  $\Lambda$  in the RS model is not much differ from the Planck scale, and the number of KK modes which is effective below  $\Lambda$  is roughly  $\sim 10^{15}$  [21]. Instead of NDA, therefore, we assume much lower

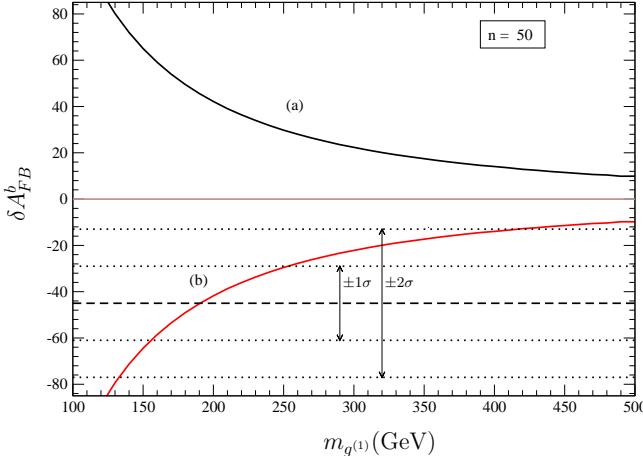


FIG. 3: The KK gluon and the KK  $b$ -quark contributions to  $\delta A_{FB}^b$  as a function of the 1st KK gluon mass,  $m_{g^{(1)}}$ . The upper and lower curves correspond to case (a)  $(\xi_L, \xi_R) = (6, 0)$  and case (b)  $(\xi_L, \xi_R) = (0, 6)$ , respectively. The horizontal dotted lines denote the allowed ranges of  $\delta A_{FB}^b$  in 1- and 2- $\sigma$  level.

In Fig. 3 we show contributions from the KK gluons and the KK  $b$ -quarks to  $\delta A_{FB}^b$  as a function of the 1st KK gluon mass. The upper and lower curves correspond to case (a)  $(\xi_L, \xi_R) = (6, 0)$  and case (b)  $(\xi_L, \xi_R) = (0, 6)$ , respectively. Note that only  $\Delta g_L^b$  receives the KK gluon contribution in (a) while  $\Delta g_R^b$  in (b). The results in the figure are obtained for the number of KK modes,  $n = 50$ . The mass of the heaviest KK mode ( $n = 50$ ) depends on the mass of 1st KK mode. For example, when  $m_{g^{(1)}} = 200$ GeV, the mass of KK gluon and  $b$ -quarks for  $n = 50$  is  $\sim 13$ TeV. The horizontal dotted lines denote the allowed range of  $\delta A_{FB}^b$  in 1- and 2- $\sigma$  level as indicated in the figure. In the 1-loop correction to  $\Delta g_\alpha^b$  (8), the sign difference comes from the  $b_\alpha$ - $b_\alpha$ - $Z$  coupling  $g_\alpha^b$  ( $\alpha = L, R$ ).

Since  $g_\alpha^b \sim I_{3b} - Q_b \sin^2 \theta_W$ , we find the relative sign of  $g_L^b$  and  $g_R^b$  is opposite. This explains that the contribution to  $\delta A_{FB}^b$  shows the opposite direction between case (a) and (b), since the coefficients of  $\Delta g_L^b$  and  $\Delta g_R^b$  have same sign as shown in eq.(3). Thus the KK gluon contribution to  $A_{FB}^b$  is favored when the KK gluon couples dominantly to  $b_R$ . In the case of  $(\xi_L, \xi_R) = (0, 6)$ , the allowed range of the 1st KK gluon mass is 150 – 250GeV in 1- $\sigma$  level (130 – 430GeV in 2- $\sigma$  level). The range of KK gluon mass shifts when the couplings  $(\xi_L, \xi_R)$  differ. A smaller value of  $\xi_R$  lowers the favored range of KK gluon mass. For example, when  $\xi_R = 4$ , the KK gluon mass which is allowed from  $A_{FB}^b$  is 90GeV-150GeV in 1- $\sigma$ .

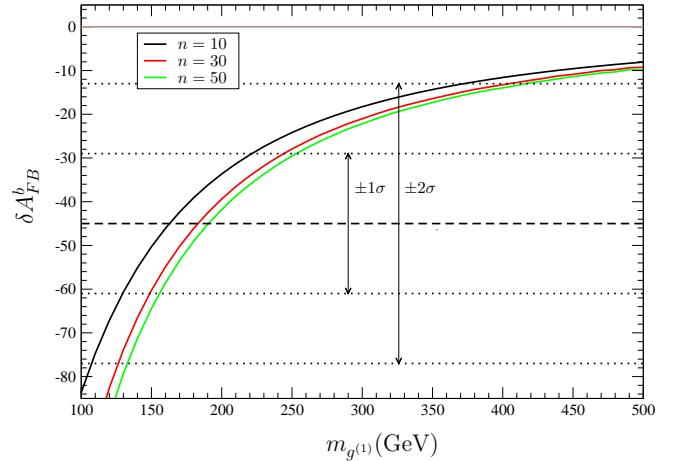


FIG. 4: The KK gluon contributions to  $\delta A_{FB}^b$  for the number of KK mode  $n = 10, 30$  and  $50$  (from upper to lower curves). The couplings are  $(\xi_L, \xi_R) = (0, 6)$ .

We have so far examined the KK gluon contribution to  $A_{FB}^b$  for the number of KK mode  $n = 50$ . The dependence of  $\delta A_{FB}^b$  on the number of KK mode is shown in Fig. 4 for  $n = 10, 30$  and  $50$ . The couplings are fixed at  $(\xi_L, \xi_R) = (0, 6)$ . We find that the difference of  $\delta A_{FB}^b$  between  $n = 10$  and  $50$  is about few 10GeV while it is few GeV between  $n = 30$  and  $50$ .

	Exp.	SM best fit	pull
$R_b$	$0.21629 \pm 0.00066$	0.21562	1.0
$A_b$	$0.923 \pm 0.020$	0.935	-0.6

TABLE I: Experimental data and the SM best fit of  $R_b$  and  $A_b$  [1]. The pull factor is defined as a deviation between data and the SM prediction normalized by the error.

The  $Zbb$  vertex correction  $\Delta g_R^b$  affects not only  $A_{FB}^b$  but also other electroweak observables for  $b$ -quark jets – for example, the partial decay rate  $R_b$  and the left-right asymmetry  $A_b$ . Here let us briefly mention about correlations between  $\Delta g_R^b$  and three observables  $A_{FB}^b, R_b, A_b$ . The experimental data and the SM prediction of  $R_b$  and  $A_b$  are summarized in Table. I. As  $A_{FB}^b$  (3),  $R_b$  and  $A_b$

can be expressed as [4]:

$$R_b(\text{NP}) = R_b(\text{SM}) - 0.78\Delta g_L^b + 0.14\Delta g_R^b, \quad (9)$$

$$A_b(\text{NP}) = A_b(\text{SM}) - 0.30\Delta g_L^b - 1.63\Delta g_R^b. \quad (10)$$

We consider  $\Delta g_L^b = 0$  ( $\xi_L = 0$ ) in the following. When the shift of  $\Delta g_R^b$  reduces the pull factor of  $A_{FB}^b$  from  $-2.8$  (SM best) to  $-1.0$ , we find that the pull factors of  $(R_b, A_b)$  from their SM best fit  $(1.0, -0.6)$  to  $(-2.4, 0.7)$ . Then  $\chi^2$  of three observables is reduced from  $9.7$  (SM best fit) to  $7.5$ . From Fig.3, the mass of 1st KK gluon which corresponds to the  $-1.0\sigma$  of  $A_{FB}^b$  data is about  $250\text{GeV}$  for  $\xi_R = 6$ . We conclude that, in a certain parameter space, the KK gluon contribution to the  $Zbb$  vertex could explain the  $A_{FB}^b$  data without affecting the current consistency of the other  $b$ -jet data,  $R_b$  and  $A_b$ .

To summarize, we have studied the KK gluon  $g^{(n)}$  in the warped extra dimension model confronts the  $A_{FB}^b$  data at the LEP experiments, which differs from the SM prediction about  $-2.8\sigma$ . We consider a scenario in which the coupling of  $g^{(1)}$  and the zero-mode  $b$ -quark could be a few times larger than the QCD coupling depending on the localization position of the bulk wavefunction of  $b$ -quark. We examined the 1-loop correction of  $Zbb$  vertex via the KK modes exchange and found that, in a certain model parameter space, the experimental data of  $A_{FB}^b$  could be explained, for example, when  $m_{g^{(1)}} \sim 150 - 250\text{GeV}$  and  $\xi_L = 0$ ,  $\xi_R = 6$ .

A few comments are in order. In our scenario the KK gluon dominantly couples to  $b_R$ . Then, the production process of  $g^{(1)}$  at hadron collider is  $b\bar{b} \rightarrow g^{(1)}$ . The production rate of  $g^{(1)}$  is, therefore, suppressed even if  $g^{(1)}$  is relatively light,  $\sim O(100\text{GeV})$ . Note that the gluon fusion process  $gg \rightarrow g^{(1)}$  is forbidden, because the zero-mode wavefunction in the fifth dimension is just a constant, and the 4D effective coupling of  $g\text{-}g\text{-}g^{(1)}$  is zero due to the orthonormality condition of gluon wavefunctions. Since the decay of  $g^{(1)}$  is possible only through  $g^{(1)} \rightarrow b\bar{b}$ , we compared the cross section  $\sigma(p\bar{p} \rightarrow g^{(1)} + X) \times \text{Br}(g^{(1)} \rightarrow b\bar{b})$  with the results given by CDF collaboration[19]. When  $(\xi_L, \xi_R) = (0, 6)$ , constraint on  $m_{g^{(1)}}$  from Tevatron is  $m_{g^{(1)}} > 157\text{GeV}$  in  $2\sigma$  level, which is consistent with the results obtained from  $A_{FB}^b$  in this paper. The other possibilities of  $g^{(1)}$  production at Tevatron are emission of  $g^{(1)}$  from  $b$  or  $\bar{b}$  quark ( $p\bar{p} \rightarrow b\bar{b}g^{(1)}$ ), and a pair production of  $g^{(1)}$  from gluon fusion,  $gg \rightarrow g^{(1)}g^{(1)}$ . After the decay of  $g^{(1)}$ , the final states are  $bbbb$  in both cases and the excess of four  $b$ -jets event may be a signal at hadron collider experiments. Also the invariant mass distributions of two  $b$ -jets  $m_{jj}$  may show a peak at  $m_{jj} = m_{g^{(1)}}$ . Therefore, the analysis of four  $b$ -jet data at Tevatron is necessary to study further constraints on  $g^{(1)}$ . It is also interesting to study these processes at LHC, and the results will be given in our forthcoming paper [20].

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[20] G.C. Cho, Y. Kanehata, N. Maru and K. Yoneyama, in preparation.

[21] The cut-off scale  $\Lambda$  in NDA for D-dimensional model is given by  $\Lambda \sim \left( (4\pi)^{D/2} \Gamma(D/2)/g_D^2 \right)^{1/(D-4)}$  [18], where  $g_D$  represents the D-dimensional gauge coupling. For the RS model, the cut-off scale is given by  $\Lambda \sim l_5/g_5^2$  with  $l_5 = 24\pi^3$  and the 5D gauge coupling is given by the 4D coupling as  $g_5 = g_4\sqrt{\pi r_c}$ . Now we count the number of KK mode. In the strong coupling limit of the 4D theory, i.e.,  $g_4^2 \sim 16\pi^2$ , If we then approximate the mass of the  $n$ -th KK mode  $m_n \simeq n\pi k \exp(-\pi kr_c)$ , the number of KK modes below the cut-off scale could be  $\Lambda/(\pi k \exp(-\pi kr_c)) \sim 10^{15}$ .